

Mirror mediation

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ABSTRACT: I show that the effective action of string compactifications has a structure that can naturally solve the supersymmetric flavour and CP problems. At leading order in the g_s and α' expansions, the hidden sector factorises. The moduli space splits into two mirror parts that depend on Kähler and complex structure moduli. Holomorphy implies the flavour structure of the Yukawa couplings arises in only one part. In type IIA string theory flavour arises through the Kähler moduli sector and in type IIB flavour arises through the complex structure moduli sector. This factorisation gives a simple solution to the supersymmetric flavour and CP problems: flavour physics is generated in one sector while supersymmetry is broken in the mirror sector. This mechanism does not require the presence of gauge, gaugino or anomaly mediation and is explicitly realised by phenomenological models of IIB flux compactifications.

KEYWORDS: Supersymmetry Breaking, Supergravity Models, Supersymmetric Standard Model, Compactification and String Models.

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1. Introduction

TeV-scale supersymmetry is one of the most promising ideas for the new physics at the weak scale that will reveal itself at the CERN Large Hadron Collider. This promise is because the presence of low-energy supersymmetry cancels the quadratic divergences in the Higgs potential, stabilising it against radiative corrections. Furthermore, supersymmetry gives a dynamical explanation for the structure of the Higgs potential through the mechanism of radiative electroweak symmetry breaking, while low scale supersymmetry is also compatible with the absence of large corrections to precision electroweak observables at LEP I.

Low energy supersymmetry is parametrised by the MSSM soft Lagrangian which specifies quantities such as squark and gaugino masses as well as trilinear scalar A-terms. One of the most significant aspects of supersymmetric phenomenology is that this Lagrangian has to take a very special form, due to the flavour and CP problems of low-energy supersymmetry. These state that generic choices of the MSSM soft Lagrangian lead to large, new and unobserved sources of flavour-changing neutral currents and CP violation. For example, if the squark masses of the first two generations are not essentially degenerate, supersymmetric contributions to $K_0 - \bar{K}_0$ mixing significantly exceed the observed rates. A

full study of the flavour physics constraints on the MSSM spectrum can be found in [1, 2], and for practical purposes these constraints can be summarised in the requirements that

$$(m^I)_{\alpha\bar{\beta}}^2 = (m^I)^2\delta_{\alpha\bar{\beta}}, \quad A_{\alpha\beta\gamma}^I = A^I Y_{\alpha\beta\gamma}.$$

That is, scalar masses of squarks and sleptons should be flavour-blind and the trilinear A-terms should be proportional to the Yukawa couplings. These constraints hold within each set of gauge-charged fields - for example, U_R and D_R squarks need not have the same mass.

A satisfactory theoretical understanding of supersymmetry breaking requires an understanding of why the soft terms should be flavour-universal. This flavour problem has been the motivation for much of the work in supersymmetric model-building. For example, gauge or gaugino mediation [3–7] solves the flavour problem by breaking supersymmetry at low energies and mediating to the observable sectors through flavour-blind gauge interactions. A different approach is that of anomaly mediation [8, 9], where universal loop effects are used to generate soft masses, although due to tachyonic slepton masses minimal anomaly mediation is not a viable phenomenological scenario. Much of the motivation for models of gauge- or anomaly-mediated supersymmetry breaking lies in the assumption that gravity-mediated supersymmetry breaking will automatically violate flavour universality. The place to test this assumption is string theory, as the prime candidate for a theory of quantum gravity.

The study of supersymmetry breaking in string theory has gone through several stages. The original work was carried out in the context of the heterotic string, with supersymmetry breaking and moduli stabilisation driven by hidden sector gaugino condensation. Although full moduli stabilisation was not possible in that context, supersymmetry breaking was analysed through a parametrisation of the goldstino direction. It was found that soft terms were flavour universal for the case of dilaton-domination, where $F^S \neq 0$ while $F^T = F^U = 0$ [10–14]. In principle, the direction of the goldstino is a dynamical quantity determined by the moduli potential. The study of the phenomenology of supersymmetry breaking has undergone a resurgence following the developments in moduli stabilisation [15–17] (for reviews of moduli stabilisation see [18–21]). These allow the goldstino direction to be explicitly computed and the structure of supersymmetric soft terms studied. This has been the subject of extensive research; recent papers studying this question include [22–45, 47, 49, 46, 48, 50–54]. The purpose of the present paper is to provide a systematic study of the necessary conditions for flavour universal soft terms and it extends arguments made in embryonic form in [40] (also see [55, 43]).

The structure of this paper is as follows. Section 2 reviews the computation of soft terms in supergravity and defines a set of sufficient conditions for the soft terms to be flavour-universal and CP-conserving. These conditions are taken as the definition of mirror mediation. They correspond to the existence of a factorisation of the hidden (moduli) fields into two sectors, with one sector generating the flavour structure and the other responsible for supersymmetry breaking. At the level of effective field theory this structure is *ad hoc*. Sections 3 and 4 are devoted to showing that the mirror mediation structure is naturally realised within type II string compactifications. In this case the susy-breaking and flavour

sectors are associated with the Kähler and complex structure moduli. Section 3 focuses on the factorisation of the moduli space and matter metrics, and section 4 on the structure of susy breaking that arises in IIB flux compactifications. An appendix studies the properties of brane intersection angles in Calabi-Yau models of intersecting brane worlds; these enter into the computation of soft scalar masses.

2. Mirror mediation

The computation of gravity-mediated¹ soft terms follows a standard structure which it is useful to review. The F-term sector of $\mathcal{N} = 1$ supergravity is specified at two derivatives by a Kähler potential, superpotential and gauge kinetic functions. These are functions of the chiral superfields, which are separated into visible and hidden sectors, C^m and Φ_i . C^m denote the matter multiplets of the MSSM. These are charged under the Standard Model gauge groups and giving them a vev reduces the rank of the gauge group. Φ_i are the hidden sector fields (the moduli). These are uncharged and may have large vevs. In addition there are the D-terms, which depend on the charged matter fields and for which it is necessary to specify the Killing vectors of the gauging.

This paper will focus on the structure of soft masses arising from F-term breaking. The principal reason for this is that within string compactification the structure of the superpotential and Kähler potential, which generate the F-term contribution to soft masses, can be discussed in a largely model-independent fashion. The moduli are normally uncharged and the relevant structure follows from simply the Calabi-Yau nature of the compactification and the geometry this induces on moduli space. In contrast, the D-term sector is much more model-dependent and is determined by the details of the charged matter content. For example, anomalous U(1)s occur frequently in string compactifications and can generate Fayet-Iliopoulos terms with significant effects on moduli stabilisation. However, to know the presence and nature of anomalous U(1)s requires knowing the details of the charged matter sector. For this reason this paper will restrict itself to the study of the F-term contributions to supersymmetry breaking and assume this to be dominant. In the case that supersymmetry breaking is dominated by D-terms, the results of this paper will be less relevant.

The full Kähler potential and superpotential can be expanded in powers of the visible sector fields,

$$K = \hat{K}(\Phi_i, \bar{\Phi}_i) + \tilde{K}_{C_\alpha \bar{C}_\beta}(\Phi_i, \bar{\Phi}_i) C^\alpha \bar{C}^\beta + (\tilde{Z}_{C_\alpha C_\beta}(\Phi_i, \bar{\Phi}_i) C^\alpha C^\beta + c.c.) + \dots, \quad (2.1)$$

$$W = \hat{W}(\Phi_i) + \mu_{\alpha\beta}(\Phi_i) C^\alpha C^\beta + Y_{\alpha\beta\gamma}(\Phi_i) C^\alpha C^\beta C^\gamma + \dots, \quad (2.2)$$

$$f = f(\Phi_i). \quad (2.3)$$

The supergravity scalar potential is

$$V = V_F + V_D, \quad (2.4)$$

¹As is well-known, the expression ‘gravity mediation’ is confusing as it does not mean ‘mediated by gravity’. It refers instead to soft terms generated by non-renormalisable contact interactions in the supergravity Lagrangian. This paper will follow convention and perpetuate this confusion.

with

$$V_F = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right), \quad (2.5)$$

where $D_i W = \partial_i W + (\partial_i K)W$. The vevs of the hidden sector moduli Φ_i are determined by the hidden sector scalar potential, which is²

$$V_F = e^{\hat{K}} \left(\hat{K}^{i\bar{j}} D_i \hat{W} D_{\bar{j}} \bar{\hat{W}} - 3|\hat{W}|^2 \right). \quad (2.6)$$

The moduli F-terms are $F^i = e^{\hat{K}/2} \hat{K}^{i\bar{j}} D_{\bar{j}} \bar{\hat{W}}$ and the gravitino mass $m_{3/2} = e^{\hat{K}/2} |\hat{W}|$. In terms of these,

$$V_F = \hat{K}_{i\bar{j}} F^i \bar{F}^{\bar{j}} - 3m_{3/2}^2. \quad (2.7)$$

The F-terms parametrise supersymmetry breaking and enter into all expressions for soft supersymmetry breaking parameters.

In gravity mediation, the vacuum is a supersymmetry-breaking minimum of (2.6) and the principal source of supersymmetry breaking is the F-terms of the hidden sector moduli. To match the observed cosmological constant, it is necessary that the F-term potential vanish, with $V_F = 0$ at the minimum. Supersymmetry breaking generates soft masses for scalars and gauginos, as well as trilinear A-terms. These arise from expanding the supergravity Lagrangian in powers of the matter fields. In particular, soft scalar masses and trilinear terms come from expanding (2.5) in powers of the matter fields C^α . The resulting soft scalar Lagrangian is

$$\mathcal{L}_{\text{soft}} = \tilde{K}_{\alpha\bar{\beta}} \partial_\mu C^\alpha \partial^\mu \bar{C}^\beta + m_{\alpha\bar{\beta}}^2 C^\alpha \bar{C}^\beta + \frac{1}{6} \left(A'_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma + c.c \right), \quad (2.8)$$

where the unnormalised soft terms are given by

$$\tilde{m}_{\alpha\bar{\beta}}^2 = (m_{3/2}^2 + V_0) \tilde{K}_{\alpha\bar{\beta}} - \bar{F}^{\bar{m}} F^n \left(\partial_{\bar{m}} \partial_n \tilde{K}_{\alpha\bar{\beta}} - (\partial_{\bar{m}} \tilde{K}_{\alpha\bar{\gamma}}) \tilde{K}^{\bar{\gamma}\delta} (\partial_n \tilde{K}_{\delta\bar{\beta}}) \right) \quad (2.9)$$

$$A'_{\alpha\beta\gamma} = e^{\hat{K}/2} F^m \left[\hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} - \left((\partial_m \tilde{K}_{\alpha\bar{\rho}}) \tilde{K}^{\bar{\rho}\delta} Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \right] \quad (2.10)$$

V_0 is the vacuum cosmological constant and will be set to zero. The gaugino masses are given by

$$M_a = F^m \frac{\partial_m f_a}{2\text{Re}(f_a)}. \quad (2.11)$$

In the case of diagonal matter metrics the soft terms can be written as

$$m_\alpha^2 = (m_{3/2}^2 + V_0) - F^{\bar{m}} F^n \partial_{\bar{m}} \partial_n \log \tilde{K}_\alpha. \quad (2.12)$$

$$A_{\alpha\beta\gamma} = F^m \left[\hat{K}_m + \partial_m Y_{\alpha\beta\gamma} - \partial_m \log(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) \right]. \quad (2.13)$$

When $\hat{K} = \hat{K}(\Phi + \bar{\Phi})$, (2.13) can be written in an instructive way

$$\hat{A}_{\alpha\beta\gamma} = F^m \frac{\partial \hat{Y}_{\alpha\beta\gamma}}{\partial \text{Re}(\Phi_m)}, \quad (2.14)$$

²We set $M_P = 1$.

where $\hat{A}_{\alpha\beta\gamma}$ and $\hat{Y}_{\alpha\beta\gamma}$ are the physical (normalised) A-terms and Yukawa couplings. In minimal flavour violation the scalar masses are flavour-universal and the trilinear terms have the same structure as the CKM matrix. This is equivalent to the requirement that the mass term $\tilde{m}_{\alpha\bar{\beta}}^2$ is proportional to the kinetic terms $\tilde{K}_{\alpha\bar{\beta}}$, and the unnormalised A-terms $A'_{\alpha\beta\gamma}$ are proportional to the superpotential Yukawa couplings $Y_{\alpha\beta\gamma}$.

It is clear that this is not the case for generic supergravity theories; for arbitrary $Y_{\alpha\beta\gamma}$ and $\tilde{K}_{\alpha\bar{\beta}}$ both conditions are violated by equations (2.10). In order for the soft terms to be flavour-universal and CP-preserving, supersymmetry breaking must decouple from flavour physics. I start by stating a set of sufficient conditions on the effective supergravity theory for gravity-mediated soft terms to be flavour-universal. The expression ‘mirror mediation’ will be used to refer to any theory satisfying these assumptions.

1. The hidden sector fields factorise into two classes, Ψ_i and χ_j .
2. The fields Ψ_i and χ_j have decoupled kinetic terms, with Ψ satisfying a reality assumption: the Kähler potential is a direct sum

$$\mathcal{K} = \mathcal{K}_1(\Psi + \bar{\Psi}) + \mathcal{K}_2(\chi, \bar{\chi}), \tag{2.15}$$

allowing the Kähler metric to be written in block-diagonal form as

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{\Psi\bar{\Psi}} & 0 \\ 0 & \mathcal{K}_{\chi\bar{\chi}} \end{pmatrix}. \tag{2.16}$$

3. The superpotential and specifically the superpotential Yukawa couplings depend only on the field χ , with no Ψ dependence:

$$Y_{\alpha\beta\gamma}(\Psi, \chi) = Y_{\alpha\beta\gamma}(\chi). \tag{2.17}$$

For the gauge kinetic functions the dependence is reversed: these depend linearly on the Ψ fields, with no χ dependence,

$$f_a(\Psi, \chi) = \sum_i \lambda_i \Psi_i. \tag{2.18}$$

4. The matter metric factorises. For any set of fields C^α, C^β carrying the same gauge charges but of different flavour, we can write

$$\mathcal{K}_{\alpha\bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi}) k_{\alpha\bar{\beta}}(\chi, \bar{\chi}) \tag{2.19}$$

with a universal dependence on Ψ . The function h is allowed to vary between fields of different gauge charges (e.g. between U_R and D_R). This also implies the factorisation of the physical Yukawa couplings,

$$\hat{Y}_{\alpha\beta\gamma}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = \underbrace{\left(\frac{e^{\mathcal{K}_1/2}}{(h_1 h_2 h_3)^{\frac{1}{2}}}(\Psi, \bar{\Psi}) \right)}_{\Psi\text{-dependent prefactor}} \times \underbrace{\left(e^{\mathcal{K}_2/2} (k^{\alpha\alpha'} k^{\beta\beta'} k^{\gamma\gamma'})^{\frac{1}{2}} Y_{\alpha'\beta'\gamma'}(\chi) \right)}_{\chi\text{-dependent flavour structure}}. \tag{2.20}$$

5. The dynamics of the vacuum is such that the Ψ fields are stabilised non-supersymmetrically and the χ fields are stabilised supersymmetrically:

$$D_{\Psi_i}W \neq 0, D_{\chi_j}W = 0. \tag{2.21}$$

Together with assumption 2, this is equivalent to the statement that $F^\Psi \neq 0, F^\chi = 0$.

These assumptions define mirror mediation. They construct two decoupled sectors Ψ and χ , such that the Ψ sector breaks supersymmetry and the χ sector generates the flavour structure. It is easy to verify that these assumptions lead to soft terms that are flavour-universal and CP-preserving. As only $F^\Psi \neq 0$ and as the matter metric factorises, we have

$$\begin{aligned} \tilde{m}_{\alpha\bar{\beta}}^2 &= (m_{3/2}^2 + V_0)\tilde{K}_{\alpha\bar{\beta}} - \bar{F}^{\bar{\Psi}_j}F^{\Psi_i} \left(\partial_{\bar{\Psi}_j}\partial_{\Psi_i}\tilde{K}_{\alpha\bar{\beta}} - (\partial_{\bar{\Psi}_j}\tilde{K}_{\alpha\bar{\gamma}})\tilde{K}^{\bar{\gamma}\delta}(\partial_{\Psi_i}\tilde{K}_{\delta\bar{\beta}}) \right) \\ &= (m_{3/2}^2 + V_0)\tilde{K}_{\alpha\bar{\beta}} - \bar{F}^{\bar{\Psi}_j}F^{\Psi_i} \left(\partial_{\bar{\Psi}_j}\partial_{\Psi_i}h(\Psi, \bar{\Psi}) - \frac{\partial_{\bar{\Psi}_j}h(\Psi, \bar{\Psi})\partial_{\Psi_i}h(\Psi, \bar{\Psi})}{h(\Psi, \bar{\Psi})} \right) k_{\alpha\bar{\beta}}(\chi, \bar{\chi}) \\ &= \left((m_{3/2}^2 + V_0)h - \bar{F}^{\bar{\Psi}_j}F^{\Psi_i} \left(\partial_{\bar{\Psi}_j}\partial_{\Psi_i}h - \frac{\partial_{\bar{\Psi}_j}h\partial_{\Psi_i}h}{h} \right) \right) (\Psi, \bar{\Psi})k_{\alpha\bar{\beta}}(\chi, \bar{\chi}) \end{aligned} \tag{2.22}$$

As the mass squares are then a constant multiple of the kinetic terms, the soft masses are flavour diagonal. Now using assumptions 3 and 4, we likewise obtain for the trilinear A-terms

$$A_{\alpha\beta\gamma} = e^{\hat{K}/2}Y_{\alpha\beta\gamma}(\chi) \left(F^\Psi \partial_{\Psi} \hat{K}(\Psi, \bar{\Psi}) - 3 \frac{F^\Psi \partial_{\Psi} h(\Psi, \bar{\Psi})}{h(\Psi, \bar{\Psi})} \right), \tag{2.23}$$

which are manifestly proportional to the Yukawa couplings.

The reality condition on the Kähler metric for the Ψ fields in assumption 2 is necessary to ensure that there is no relative phase between different A-terms, with the phase of all A-terms being set by that of the goldstino F-term. The linearity condition in assumption 3 likewise ensures that the gaugino mass phases are universal and aligned with those of the A-terms. The linearity condition of equation (2.18) did not enter into the expression for soft masses in (2.22), but is necessary to ensure that gaugino mass phases are universal.³ If a μ -term is generated through the Giudice-Masiero mechanism [57], then the reality condition also ensures that the phase of the μ term aligns with that of the A-terms and gaugino masses.

If a theory exhibits mirror mediation, the soft terms it generates are automatically flavour-universal and CP-preserving. Formulated within effective field theory, the assumptions that enter mirror mediation are *ad hoc*: their entire purpose is to ensure flavour universality. The purpose of this paper is to point out that in string theory the mirror mediation structure is realised naturally and occurs in large classes of string compactifications.

³The form of equation (2.18) will receive additional threshold corrections, but the detailed study of these is beyond the scope of this paper, which aims to study the leading order structure.

3. Mirror mediation in string theory

We wish to relate the structure of mirror mediation to that appearing in string compactifications. In string compactifications the hidden sector of effective field theory should be identified with the moduli of the compactification. These are associated with the extra-dimensional geometry and enter into expressions for the gauge and Yukawa couplings of the low-energy theory. To preserve low-energy supersymmetry, string theory should be compactified on a Calabi-Yau manifold. In this case the Calabi-Yau geometry naturally provides two main classes of moduli, Kähler and complex structure moduli. The Kähler moduli describe the size of the Calabi-Yau and the complex structure moduli the shape. In terms of the Hodge numbers of the Calabi-Yau there are $h^{1,1}$ Kähler moduli and $h^{2,1}$ complex structure moduli. We will identify the two sectors Ψ and χ required for mirror mediation with the Kähler and complex structure moduli. Which way the identification goes will depend on whether the string theory involved is IIA or IIB.⁴

3.1 Factorisation of moduli space

The second assumption of mirror mediation is that the moduli space factorises. In pure $\mathcal{N} = 2$ type II string compactifications, this factorisation is exact and ensured by mirror symmetry. In $\mathcal{N} = 1$ type II orientifold compactifications, the factorisation still exists at leading order, being broken by subleading corrections. For IIA string theory, the classical Kähler potential (at leading order in the g_s and α' expansions) is [56]

$$\mathcal{K} = -\ln(\mathcal{V}) - 2\ln\left(\int \text{Re}(C\Omega) \wedge *\text{Re}(C\bar{\Omega})\right). \quad (3.1)$$

C is a compensator field that incorporates the dilaton dependence. For IIB string theory, we have

$$\mathcal{K} = -2\ln(\mathcal{V}) - \ln\left(i \int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S}). \quad (3.2)$$

In both cases the dependence of the Kähler potential on the Kähler moduli T_i and the complex structure moduli U_i factorises. As chiral superfields, the definitions of ‘Kähler moduli’ and ‘complex structure moduli’ differ in IIA and IIB:

$$\begin{aligned} \text{IIB(D3/D7)} : \quad T &= e^{-\phi}\text{Vol}(\Sigma_4) + iC_4, & U &= \int_{\Sigma_3} \Omega, \\ \text{IIA} : \quad T &= \text{Vol}(\Sigma_2) + iB_2, & U &= e^{-\phi}\text{Vol}(\Sigma_3) + iC_3. \end{aligned} \quad (3.3)$$

Σ_k refers to a cycle in the Calabi-Yau of dimensionality k and the RR forms are understood to be integrated over these cycles. The different definitions of the moduli explain the

⁴The mirror mediation structure relies on the existence of two separate classes of moduli, which at leading order decouple. In weakly coupled IIA, IIB and heterotic models, the Kähler and complex structure moduli provide these two separate classes. There are however regimes in which this breaks down. For example, in the case of M-theory, which is the strong coupling limit of IIA string theory, there is only one class of moduli, which are associated with the geometry of 3-cycles. In this case all moduli are on an equal footing and the mirror mediation structure is not possible.

apparent different factors of the volume in (3.1) and (3.2). In toroidal examples the Kähler potential has a simple expression for both IIA and IIB models,

$$\mathcal{K} = -\ln(S+\bar{S}) - \ln\left((T_1+\bar{T}_1)(T_2+\bar{T}_2)(T_3+\bar{T}_3)\right) - \ln\left((U_1+\bar{U}_1)(U_2+\bar{U}_2)(U_3+\bar{U}_3)\right) \quad (3.4)$$

From (3.1) and (3.2) we see that the Kähler potential admits a factorised form, consistent with assumption 2. The Kähler metric can be written as

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{T\bar{T}} & 0 & 0 \\ 0 & \mathcal{K}_{U\bar{U}} & 0 \\ 0 & 0 & \mathcal{K}_{S\bar{S}} \end{pmatrix}. \quad (3.5)$$

The different classes of moduli represent distinct sectors, with kinetic terms that do not mix with each other.

The factorisation is broken by subleading corrections, which lead to non-vanishing values for $\mathcal{K}_{T\bar{U}}$ and $\mathcal{K}^{T\bar{U}}$. The discussion here will focus on the IIB case, but analogous results will hold in IIA. One way factorisation can be broken is through the presence of D3 branes. Such branes will generically be present in IIB flux compactifications, as it is necessary to cancel the tadpoles arising from O3-planes. It is not possible to realise a chiral matter spectrum on D3 branes away from orbifold fixed points (as the local theory of D3 branes is $\mathcal{N} = 4$ supersymmetric), but this is not relevant here: the D3 branes are used as an illustration of one way in which factorisation can break down. For example, we can consider the toroidal $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold in the presence of D3 branes. The Kähler potential is then (e.g. see [58])

$$\mathcal{K} = -\sum_{I=1}^3 \log \left[(T_i + \bar{T}_i)(U_i + \bar{U}_i) + \frac{1}{8\pi}(A_i + \bar{A}_i)^2 \right]. \quad (3.6)$$

Here A_i are brane moduli that parametrise the position of a D3 brane on the torus i . The sum is over each of the three tori. It is straightforward to compute the resulting Kähler metric and its inverse. The metric is a direct sum of three terms, one for each torus. Letting I index the torus, we have

$$\mathcal{K}_I^{-1} = \begin{pmatrix} (T_I + \bar{T}_I)^2 & -\frac{(A_I + \bar{A}_I)^2}{8\pi} & (A_I + \bar{A}_I)(T_I + \bar{T}_I) \\ -\frac{(A_I + \bar{A}_I)^2}{8\pi} & (U_I + \bar{U}_I)^2 & (A_I + \bar{A}_I)(U_I + \bar{U}_I) \\ (A_I + \bar{A}_I)(T_I + \bar{T}_I) & (A_I + \bar{A}_I)(U_I + \bar{U}_I) & \left(\frac{(A_I + \bar{A}_I)^2 - 8\pi(T_I + \bar{T}_I)(U_I + \bar{U}_I)}{2} \right) \end{pmatrix} \quad (3.7)$$

In the limit that the cycle sizes are large, the T and U sectors remain factorised. The figure of merit for this is the ratio, η , of $(\mathcal{K}^{-1})^{T\bar{U}}$ and $(\mathcal{K}^{-1})^{T\bar{T}}$. These are given by

$$\mathcal{K}^{T\bar{T}} = (T + \bar{T})^2, \quad \mathcal{K}^{U\bar{T}} = -\frac{(A + \bar{A})^2}{8\pi}, \quad \eta = -\frac{(A + \bar{A})^2}{8\pi(T + \bar{T})^2}.$$

The reason why η represents the figure of merit is that it gives the cross-coupling induced from F^T to F^U given that $D_T W \neq 0$ and $D_U W = 0$. As

$$F^T = e^{K/2} K^{T\bar{I}} D_{\bar{I}} W = e^{K/2} K^{T\bar{T}} D_{\bar{T}} W,$$

$$F^U = e^{K/2} K^{U\bar{I}} D_{\bar{I}} W = e^{K/2} K^{U\bar{T}} D_{\bar{T}} W,$$

η measures the ratio F^U/F^T , namely the extent to which cross-couplings induce F-terms in the ‘wrong’ sector. This cross-coupling is suppressed by a factor $(T + \bar{T})^{-2}$, and vanishes in the large-volume limit.

More generally, in the presence of a D3 brane with position moduli ϕ^i the T fields are redefined as [59]

$$T_\alpha = \text{Vol}(\Sigma_4) + iC_4 + (\omega_\alpha)_{i\bar{j}} \phi^i \left(\bar{\phi}^{\bar{j}} - \frac{i}{2} \bar{z}^a (\chi_a)_{\bar{l}}^{\bar{j}} \phi^{\bar{l}} \right), \quad (3.8)$$

where ω_α is the 2-form associated with T_α evaluated at the brane locus, and \bar{z}^a and χ_a are the complex structure moduli and associated $(2, 1)$ forms. For a single Kähler modulus model, the resulting Kähler potential $\mathcal{K} = -3 \ln(\text{Vol}(\Sigma_4))$ can be written as

$$K = \mathcal{K}_{U\bar{U}} - 3 \ln(T + \bar{T}) + \frac{(\phi\bar{\phi} + \phi\phi f(U, \bar{U}))}{T + \bar{T}} + \dots \quad (3.9)$$

This gives

$$K_{T\bar{T}} = \frac{3}{(T + \bar{T})^2} - \frac{2(\phi\bar{\phi} + \phi\phi f(U, \bar{U}))}{(T + \bar{T})^3}, \quad K_{T\bar{U}} = -\frac{\phi\phi\partial_{\bar{U}}f}{(T + \bar{T})^2}, \quad K_{U\bar{U}} = \mathcal{K}_{U\bar{U}} + \frac{\phi\phi\partial_{\bar{U}}\partial_U f}{(T + \bar{T})}.$$

Focusing on the T and U components, this gives

$$\mathcal{K}^{-1} = \begin{pmatrix} \frac{(T+\bar{T})^2}{3} + \mathcal{O}(T + \bar{T}) & -\frac{\phi\phi\partial_{\bar{U}}f}{3\mathcal{K}_{U\bar{U}}} \\ -\frac{\phi\phi\partial_{\bar{U}}f}{3\mathcal{K}_{U\bar{U}}} & \mathcal{K}^{U\bar{U}} + \mathcal{O}\left(\frac{1}{T+\bar{T}}\right) \end{pmatrix}. \quad (3.10)$$

We again have $\eta \sim (T + \bar{T})^{-2}$, and the moduli spaces are approximately factorised at large volume: non-zero F^U cannot be induced from non-zero F^T . The restoration of factorisation at large volumes is consistent with intuition. The breaking of factorisation occurred because the D3-brane positions mix with the Kähler moduli controlling 4-cycle volume. This occurs because the D3 branes back-react on the geometry, and this backreaction alters the 4-cycle sizes [60], which enters into the holomorphic chiral superfields. However, the larger the volume the more the effect of D3 brane back-reaction is diluted by the volume of the compact space and the less effect it has on the moduli space factorisation.

Another source of corrections that violate factorisation are loop corrections. These can arise from either D3 or D7 branes. We focus on the corrections due to D7 branes since these are more generic - it is always possible to avoid including D3 branes by saturating the O3 tadpole with 3-form flux. Loop corrections to the Kähler potential in torodial backgrounds have been computed in [61]. For convenience we fix $T_1 = T_2 = T_3$, $U_1 = U_2 = U_3$. The loop-corrected Kähler potential due to the presence of D7 branes is then

$$K = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T}) - 3 \ln(U - \bar{U}) + \frac{3}{256\pi^6} \frac{\mathcal{E}_2(0, U)}{(T + \bar{T})^2} \quad (3.11)$$

Here

$$\mathcal{E}_2(0, U) = - \sum_{(m,n) \neq (0,0)} \frac{1920(U - \bar{U})^2}{(n + mU)^2(n + m\bar{U})^2}$$

This Eisenstein series has the property that

$$\partial_U \partial_{\bar{U}} \mathcal{E}_2(0, U) = -\frac{2}{(U - \bar{U})^2} \mathcal{E}_2(0, U).$$

The Kähler metric is

$$\mathcal{K} = \begin{pmatrix} \frac{3}{(T+\bar{T})^2} & 0 \\ 0 & \frac{-3}{(U-\bar{U})^2} \end{pmatrix} + 3 \begin{pmatrix} \frac{6}{256\pi^6} \frac{\mathcal{E}_2(0,U)}{(T+\bar{T})^4} & \frac{-2}{256\pi^6} \frac{\partial_{\bar{U}} \mathcal{E}_2(0,U)}{(T+\bar{T})^3} \\ \frac{-2}{256\pi^6} \frac{\partial_{\bar{T}} \mathcal{E}_2(0,U)}{(T+\bar{T})^3} & \frac{-2}{256\pi^6} \frac{\mathcal{E}_2(0,U)}{(U-\bar{U})^2 (T+\bar{T})^2} \end{pmatrix}. \quad (3.12)$$

This gives

$$K^{T\bar{T}} = \frac{(T + \bar{T})^2}{3} + \dots, \quad (3.13)$$

$$K^{U\bar{T}} = \frac{1}{3} \frac{1}{128\pi^6} \frac{1}{(T + \bar{T})} \left(\sum_{(n,m) \neq (0,0)} \frac{2(U - \bar{U})^3 \times 1920}{(n + mU)^3 (n + m\bar{U})} \right). \quad (3.14)$$

The ratio

$$\eta = \frac{K^{U\bar{T}}}{K^{T\bar{T}}} = \frac{1}{128\pi^6} \frac{1}{(T + \bar{T})^3} \sum_{(n,m) \neq (0,0)} \frac{2(U - \bar{U})^3 \times 1920}{(n + mU)^3 (n + m\bar{U})}.$$

The breaking of factorisation due to the loop corrections goes as $(T + \bar{T})^{-3}$ at large cycle volumes.

A final source of factorisation-violating corrections to the Kähler potential are those arising from higher α' corrections. Fluxes couple to complex structure moduli, and so α' -corrections involving the fluxes will therefore lead to corrections to the Kähler potential that will mix Kähler and complex structure moduli, violating factorisation. For example, the 10d α'^3 correction

$$\frac{1}{\alpha'^4} \int d^{10}x (\mathcal{R} + \alpha'^3 (G_3^2 \mathcal{R}^3 + c.c.))$$

should lead to a correction to K . The tree-level moduli kinetic terms arise from the dimensional reduction of the \mathcal{R} term. Using simple scaling arguments (flux is quantised on 3-cycles, so $G_3^2 \sim N^2/\mathcal{V}$, while $\mathcal{R} \sim \mathcal{V}^{-1/3}$), it follows that corrections due to the $G_3^2 \mathcal{R}^3$ term are suppressed by $\mathcal{V}^{-5/3}$ compared to the tree level kinetic terms. At large volume, the violation of factorisation due to such terms will therefore be very small.

Let us summarise the results of this section. At leading order, the string theory moduli space factorises. The factorisation is broken by loop corrections, α' corrections and by the presence of branes. In all these cases the breaking of factorisation is suppressed at large volume: in the limit that the volume increases while all other fields are held constant factorisation is restored.

3.2 Factorisation of superpotential Yukawa couplings

The third requirement of mirror mediation is that the superpotential Yukawa couplings depend only on the χ (flavour) sector while the gauge couplings depend on the susy-breaking Ψ sector.

An important feature of the definitions of moduli superfields in string theory is the presence of Peccei-Quinn symmetries. In IIB these correspond to $T \rightarrow T + i\epsilon$, $S \rightarrow S + i\epsilon$, with U not having a shift symmetry,⁵ whereas in IIA all three sets of moduli have shift symmetries, $S \rightarrow S + i\epsilon$, $T \rightarrow T + i\epsilon$, $U \rightarrow U + i\epsilon$. These shift symmetries arise because the imaginary parts originate from axionic terms: $\text{Im}(T)_{\text{IIB}} = C_4, \text{Im}(S)_{\text{IIB}} = C_0, \text{Im}(S)_{\text{IIA}} = C_3, \text{Im}(T)_{\text{IIA}} = B_2, \text{Im}(U)_{\text{IIA}} = C_3$. Axions only have topological couplings and so perturbation theory (which is an expansion about topologically trivial states) is insensitive to them. The axionic shift symmetries can be broken only by effects non-perturbative in the worldsheet (α') or spacetime (g_s) expansions.⁶

In IIB, the T shift symmetry is unbroken in both space-time and world-sheet perturbation theory. It can be broken by D3-instantons. The S shift symmetry is unbroken in space-time perturbation theory and can be broken by D(-1)-instantons. For IIA models, the T shift symmetry is unbroken in world-sheet perturbation theory and is broken by worldsheet instantons, whereas the S and U symmetries are unbroken in both spacetime and worldsheet perturbation theory and can only be broken by D2-instantons.

Up to such non-perturbative effects the Peccei-Quinn symmetries remain exact. This strongly constrains the moduli that can enter the superpotential and in particular the superpotential Yukawa couplings. The requirement that the superpotential be both holomorphic in the moduli and preserve the Peccei-Quinn symmetries eliminates any perturbative dependence on moduli having the Peccei-Quinn symmetry. Assuming the string coupling to be small, we then know that within spacetime perturbation theory

$$Y_{\alpha\beta\gamma, \text{IIA}}(S, T, U) = Y_{\alpha\beta\gamma}(T), \tag{3.15}$$

$$Y_{\alpha\beta\gamma, \text{IIB}}(S, T, U) = Y_{\alpha\beta\gamma}(U). \tag{3.16}$$

The Yukawa couplings depend only on the T -moduli in IIA and on the U -moduli in IIB. This structure matches onto the third assumption required for mirror mediation. We can also now identify the Ψ sector with the Kähler moduli in IIB and with the complex structure moduli in IIA, and vice-versa for the χ sector.⁷

This structure of Yukawa couplings fits with the explicit computations (we will discuss these further below). In IIB compactifications, Yukawa couplings have essentially classical

⁵In the limit of large complex structure the IIB U moduli do possess a shift symmetry, which is mirror symmetric to the shift symmetry of the IIA Kähler moduli that is always broken by world-sheet instantons. Note the world-sheet instantons which break the shift symmetry of the IIA Kähler moduli are classical effects in g_s , and so the shift symmetry of the Kähler moduli is broken at the classical level in space-time perturbation theory.

⁶Some axionic symmetries can also be broken by fluxes; for example the IIB dilaton shift symmetry and the IIA Kähler moduli shift symmetries can be broken by fluxes. Metric fluxes can also be said to lift shift symmetries, but as these correspond to a change in the topology of the compactification and thus the number of non-contractible cycles, it is not clear that ‘lifting’ is the appropriate expression. The breaking of shift symmetries by fluxes, where it occurs, will not affect the arguments given here.

⁷Similar Peccei-Quinn symmetries constraining the perturbative appearance of T and S moduli can be found in heterotic string theory. The strongly coupled heterotic string is related by dualities to type I (that is, type IIB orientifold) models. This suggests that the mirror mediation structure may also exist in heterotic models. Dilaton domination can be seen as an example of this, where the susy breaking sector is viewed as the (S, T) moduli and the U moduli as the flavour sector.

origins. Chiral fermions arise from the reduction of the DBI/super Yang-Mills actions in the presence of magnetic flux. The fermion modes and wavefunctions are found by solving the higher-dimensional Dirac equation in the presence of magnetic flux. The Dirac equation depends on the complex geometry - i.e. the complex structure - of the Calabi-Yau. The Yukawa couplings arise from the classical overlap of these wavefunctions, and are non-vanishing even in the field theory limit $g_s \rightarrow 0, \alpha' \rightarrow 0$. This is manifest from the form of (3.15): the Yukawa couplings depend on the U moduli, which enter into neither the g_s nor α' expansions.

In IIA, chirality arises through the pointlike intersection of D6-branes in extra dimensions. Each intersection point gives rise to a chiral fermion. Matter is localised at the intersection point and so there are no classical couplings between different species. Due to the spatial separation, all Yukawa couplings must arise nonperturbatively. The Yukawa couplings are generated by worldsheet instantons, which are non-perturbative in the α' expansion. They depend only on the Kähler moduli and appear as $e^{-2\pi T_i}$, breaking the T Peccei-Quinn symmetry.

The superpotential factorisation of equations (3.15) and (3.16) is broken by brane instantons, effects which are non-perturbative in the string coupling. These allow a dependence on $T(U)$ moduli to appear in the IIB (IIA) superpotential or Yukawa couplings. Such effects are non-perturbative in both the g_s and α' expansions. In the limit that either the string coupling is small or the volume is large such effects are therefore exponentially suppressed. The fields entering the exponents are furthermore the same fields that define the high-scale Standard Model gauge couplings. As these couplings are small instanton effects are expected to be insignificant: in moduli stabilised models, it is often the case that $e^{-T} \sim \frac{m_{3/2}}{M_P} \lesssim 10^{-15}$.

Moduli also have different roles in determining the gauge couplings. Gauge couplings correspond to the volumes of cycles wrapped by branes. For IIB with matter on D7 branes, which is the phenomenologically interesting case,

$$f_a = T_a + h_a(F)S. \tag{3.17}$$

$h_a(F)$ depends on the magnetic fluxes present on the branes. For IIA models with intersecting D6-branes,

$$f_a = U_a, \tag{3.18}$$

where U_a is the modulus (either complex structure or dilaton) controlling the volume of the 3-cycle wrapped by the D6 branes.

It then follows that the third assumption of mirror mediation is satisfied in string compactifications, as the superpotential Yukawa couplings depend only on one class of moduli whereas the gauge couplings depend on the other class.

3.3 Factorisation of physical Yukawa couplings

The fourth requirement of mirror mediation is that the matter metrics - and hence the physical Yukawa couplings - factorise, with the χ sector determining the flavour structure and the Ψ sector serving only as an overall normalisation.

To illustrate the different roles played by Kähler and complex structure moduli, we write down the full expressions for the physical Yukawa couplings for toroidal compactifications. In IIB compactifications the classical Yukawa couplings - those applicable at leading order in the g_s and α' expansions - can be computed in field theory. This procedure was carried out in the paper [62], whose authors dimensionally reduced the ten-dimensional Yang-Mills action on a toroidal background in the presence of magnetic flux. This is related by T-duality to D3-D7 systems. The following expression was obtained for the physical Yukawa couplings:

$$Y_{ijk} = \underbrace{\frac{1}{\sqrt{\mathcal{V}}}}_{T\text{-dependence}} \underbrace{g_s \left(\prod_{r=1}^3 2\text{Im}U^r \right)^{\frac{1}{4}} \left| \frac{\tilde{I}_1^r \tilde{I}_2^r}{\tilde{I}_1^r + \tilde{I}_2^r} \right|^{1/4} \vartheta \left[\begin{matrix} \delta_{ijk}^r \\ 0 \end{matrix} \right]}_{U\text{-dependence}} (0, U^r | I_{ab}^r I_{bc}^r I_{ca}^r) \quad (3.19)$$

The quantities I^r, \tilde{I}^r are integers related to flux quantisation, while $\delta_{ijk}^r = \frac{i^r}{I_{ab}^r} + \frac{j^r}{I_{ca}^r} + \frac{k^r}{I_{bc}^r}$. The flavour structure of the Yukawa couplings appear through the ϑ -function. This depends on the U -moduli whereas the Kähler moduli appear as an overall flavour-independent normalisation. The Kähler moduli can only affect the overall scale of the Yukawa couplings and cannot affect the texture and relative hierarchies of the couplings.

For toroidal compactifications the Yukawas can also be computed in the full string theory. The stringy computation for IIB with magnetised D9-branes gives [63, 64, 20, 65, 66]

$$Y_{ijk}^{\text{IIB}} = \overbrace{\frac{1}{\mathcal{V}^{1/4}} \prod_{r=1}^3 \sigma_{abc} \left(\frac{\Gamma(1 - \frac{1}{\pi}\phi_{ab}^r)\Gamma(1 - \frac{1}{\pi}\phi_{ca}^r)\Gamma(\frac{1}{\pi}(\phi_{ab}^r + \phi_{ca}^r))}{(2\pi)^3\Gamma(\frac{1}{\pi}\phi_{ab}^r)\Gamma(\frac{1}{\pi}\phi_{ca}^r)\Gamma(1 - \frac{1}{\pi}(\phi_{ab}^r + \phi_{ca}^r))} \right)^{1/4}}^{\text{T-dependence}} \underbrace{\times e^{\phi_{10}/2} (U^r)^{1/4} \vartheta \left[\begin{matrix} \delta_{ijk}^r \\ 0 \end{matrix} \right]}_{\text{U-dependence}} (0; U^r I_{ab}^r I_{bc}^r I_{ca}^r). \quad (3.20)$$

The angles ϕ_{ab}^r satisfy $\phi_{ab}^r + \phi_{bc}^r + \phi_{ca}^r = 0$ and are given by

$$\phi_{ab}^r = \arctan\left(\frac{f_b^r}{t_2^r}\right) - \arctan\left(\frac{f_a^r}{t_2^r}\right), \quad (3.21)$$

with t_2 2-cycle volumes. In the small-angle dilute flux limit, $t_2 \gg f_a, f_b$, this expands as

$$\phi_{ab}^r = \left(\frac{f_b^r - f_a^r}{t_2^r}\right) - \frac{1}{3} \left(\left(\frac{f_b^r}{t_2^r}\right)^3 - \left(\frac{f_a^r}{t_2^r}\right)^3 \right) + \frac{1}{5} \left(\left(\frac{f_b^r}{t_2^r}\right)^5 - \left(\frac{f_a^r}{t_2^r}\right)^5 \right) + \dots \quad (3.22)$$

The expansion of the gamma functions gives

$$\left(\frac{\Gamma(1 - \frac{1}{\pi}\phi_{ab}^r)\Gamma(1 - \frac{1}{\pi}\phi_{ca}^r)\Gamma(\frac{1}{\pi}(\phi_{ab}^r + \phi_{ca}^r))}{(2\pi)^3\Gamma(\frac{1}{\pi}\phi_{ab}^r)\Gamma(\frac{1}{\pi}\phi_{ca}^r)\Gamma(1 - \frac{1}{\pi}(\phi_{ab}^r + \phi_{ca}^r))} \right)^{1/4} = \frac{1}{\pi} \frac{\phi_{ab}^r \phi_{ca}^r}{\phi_{ab}^r + \phi_{ca}^r} - \frac{2}{\pi^4} (\phi_{ac}^r \phi_{ca}^r)^2 + \dots \quad (3.23)$$

In the limit that the angles $\phi_{ab} \rightarrow 0$, equation (3.20) reduces to (3.19). Note however that eq. (3.20) retains the factorised form to all orders in α' . This also shows that the breakdown of the single classical T -scaling occurs at order $\left(\frac{f}{t}\right)^2$ in the dilute flux expansion - this is an $\mathcal{O}(\alpha'^2)$ effect.⁸ The breakdown of the classical scaling can be understood from the fact that the higher dimensional action is actually the DBI action rather than the super Yang-Mills.

In the T-dual picture with intersecting D6-branes, the angles ϕ_{ab}^r are the physical intersection angles between different branes. The IIA result for intersecting D6-branes is the mirror-symmetric form of the above in which T and U are interchanged. It takes the form [67, 68]

$$Y_{ijk}^{\text{IIA}} = e^{\Phi_4/2} \prod_{r=1}^3 \sigma_{abc}(t^r)^{1/4} \left(\frac{\Gamma(1 - \frac{1}{\pi}\phi_{ab}^r)\Gamma(1 - \frac{1}{\pi}\phi_{ca}^r)\Gamma(\frac{1}{\pi}(\phi_{ab}^r + \phi_{ca}^r))}{(2\pi)^3\Gamma(\frac{1}{\pi}\phi_{ab}^r)\Gamma(\frac{1}{\pi}\phi_{ca}^r)\Gamma(1 - \frac{1}{\pi}(\phi_{ab}^r + \phi_{ca}^r))} \right)^{1/4} \times \vartheta \begin{bmatrix} \delta_{ijk}^r \\ 0 \end{bmatrix} (0; t^r I_{ab}^r I_{bc}^r I_{ca}^r). \tag{3.24}$$

Here $\Phi_4 = \phi_{10} - \frac{1}{2} \ln \mathcal{V}$ is the 4-dimensional dilaton and the t^r of IIA refer to 2-cycle volumes. In IIA string theory, the angles ϕ_{ab} depend on the complex structure moduli rather than the Kähler moduli as in IIB.

From the above formulae we see that the Yukawa couplings do admit a factorised form. The flavour-dependent part is encoded in the ϑ -functions and depends on the complex structure in IIB and on the Kähler moduli in IIA. These involve exponentials, which could naturally lead to the mass hierarchies within the Standard Model.

In addition, there is a universal flavour-independent normalisation prefactor. This depends on the Kähler moduli in IIB and on the complex structure moduli in IIA. In the dilute flux limit in which $\phi_{ab} \rightarrow 0$ for all ϕ , the Kähler moduli appear simply as an overall prefactor with a single power. This is illustrated in the field theory limit through the formula (3.19).

The factorisation of the Yukawa couplings has a simple field theory origin which is most easily understood in the IIB formalism. This also allows an understanding of why factorisation extends beyond the toroidal case. In IIB compactifications, chirality arises due to the presence of magnetic fluxes on brane world-volumes. Chiral fermions arise as zero modes of the Dirac equation in the presence of magnetic flux. The relative magnetic flux distinguishes two brane stacks and leads to bifundamental fermions. Different flavours correspond to different zero modes. The Dirac equation is

$$\Gamma^M D_M \lambda = \Gamma^M \left(\partial_M \lambda + \frac{1}{4} \omega_M^{kl} \Sigma_{kl} \lambda + [A_M, \lambda] \right) = 0. \tag{3.25}$$

The spin connection ω_M^{kl} is defined through the vielbein e_a^M , with $g^{MN} = e_a^M \eta^{ab} e_b^N$, $\Gamma^M = e_a^M \gamma^a$ with γ^a the flat-space Dirac matrices, and $\Sigma_{kl} = \frac{1}{4} \gamma_{[k} \gamma_{l]}$. Then

$$\omega_M^{ab} = \frac{1}{2} g^{RP} e_R^{[a} \partial_{[M} e_{P]}^{b]} + \frac{1}{4} g^{RP} g^{ST} e_R^{[a} e_T^{b]} \partial_{[S} e_{P]}^c e_M^d \eta_{cd}. \tag{3.26}$$

⁸This also implies that in the field theory limit the supersymmetry condition $\phi_1 + \phi_2 + \phi_3 = 0$ does not place constraints on the Kähler moduli.

Under rescalings $g \rightarrow \lambda g$ the spin connection is unchanged. Likewise, the gauge field A_i is specified in terms of the complex coordinates of the space and is unaffected by rescalings of the metric. Any zero mode of (3.25) is therefore unaltered by a metric rescaling. As the texture of Yukawa couplings comes from the overlap of zero modes, these are also unaffected by metric rescalings.

Rescalings do however affect the normalisation of the zero modes. To have canonical kinetic terms, the zero modes must satisfy

$$\int_{\Sigma} \sqrt{g} |\psi|^2 = 1. \tag{3.27}$$

where Σ is the submanifold on which they are supported. The wavefunctions must then be normalised as $\psi_N \sim \frac{1}{\sqrt{Vol(\Sigma)}} \psi_0$. This normalisation depends only on the volume of Σ and is therefore flavour-independent.

Physical Yukawa couplings arise from the triple overlap of three normalised wavefunctions (due to supersymmetry the bosonic zero modes have the same functional form as their fermionic partners). Metric rescalings enter the physical Yukawas through the normalisation condition (3.27), which is flavour-independent. This gives a universal (i.e. factorised) form with respect to the metric modes which rescale cycle volumes: these modes are the Kähler moduli.

Specifically, in the case that all chiral matter arises from branes wrapping the same cycle, the Yukawa couplings descend from the term

$$\int_{\Sigma} \sqrt{g} \Gamma^{\mu} D_{\mu} \psi \rightarrow \int_{\Sigma} \sqrt{g} \bar{\psi} (\Gamma^M A_M) \psi.$$

This applies both to D9 branes and to models of branes at (resolved) singularities. Canonical normalisation of the kinetic terms requires (up to numerical factors)

$$\int_{\Sigma} \sqrt{g} (\bar{\psi} \psi)^2 = 1, \int_{\Sigma} \sqrt{g} (\Gamma^M A_M)^2 = 1,$$

and so putting all factors of the cycle volumes together the physical Yukawa couplings have a universal scaling of $Vol(\Sigma) \times \left(\frac{1}{\sqrt{Vol(\Sigma)}}\right)^3 = \frac{1}{\sqrt{Vol(\Sigma)}}$. This scaling is independent of the detailed flavour structure. This is precisely the behaviour seen in equation (3.19), where all chiral matter arises from wrapped D9-branes. However this argument does not rely on a toroidal background and applies equally well to the Calabi-Yau case.⁹

3.4 Factorisation of matter metrics

In the expression for the soft masses and A-terms it is not just the Yukawa couplings but actually the matter metrics that enter the physical Yukawa couplings. These are not so easy to compute directly through dimensional reduction. However, in the field theory limit the modular weights of the matter metrics can be inferred indirectly from the scalings of

⁹This volume scaling means that D9 branes, or indeed any brane wrapping a cycle scaling with the overall volume, cannot be used to realise the Standard Model within large volume compactifications.

the physical Yukawa couplings. The supergravity structure implies the physical Yukawa couplings can be written as

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}. \tag{3.28}$$

The Kähler potential \hat{K} is known. We have seen in section 3.2 that the combination of holomorphy and shift symmetries implies certain moduli cannot appear in $Y_{\alpha\beta\gamma}$, and so if the modular scaling of the physical Yukawa coupling can be computed then the modular weights of the corresponding kinetic terms can be inferred. This logic was used in [69] to compute the modular weights for localised chiral D7-D7 matter in the large volume models. For example, in the case above where all chiral matter arises from branes wrapping the same cycle, we can deduce that

$$e^{\hat{K}/2} \frac{1}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}} \sim \frac{1}{\text{Vol}(\Sigma)^{\frac{1}{2}}}. \tag{3.29}$$

The derivation of this only uses the region on which the wavefunction (i.e. the cycle Σ) is supported and not the particular form of the wavefunction. Each index (α , β and γ) of the Yukawa couplings corresponds to a different set of gauge-charged fields. If a different flavour is used in computing the Yukawa couplings, then this corresponds to replacing the index α by α' (e.g. using the top quark rather than the up quark). As fields with identical gauge charges are supported on the same cycle the same volume scaling occurs, which implies the two flavours must, under metric rescalings, have the same modular weights in the field theory limit.

The Peccei-Quinn symmetries $T \rightarrow T + i\epsilon$ also enforce the reality conditions that are present in assumptions 2 and 4 of mirror mediation. The requirement that the shift symmetry be unbroken in perturbation theory implies that there can be no explicit dependence on the (imaginary) axionic components of T : T can only appear in the Kähler metrics as $(T + \bar{T})$.

The matter metrics have been explicitly computed in toroidal backgrounds using string worldsheet analyses. The intersection of any two D6 branes is invariantly characterised by three angles θ_1 , θ_2 , θ_3 , with $\theta_1 + \theta_2 + \theta_3 = 0$. In [70] the expression for the matter metric for the chiral field on the brane intersection is given by

$$K_{ij}^{ab} = \delta_{ij} S^{-1/4} \prod_{I=1}^3 U_I^{-1/4} T_I^{-\left(\frac{1}{2} \pm \frac{1}{2} \text{sign}(I_{ab}) \theta_{ab}^1\right)} \left(\frac{\Gamma(\theta_{ab}^1) \Gamma(\theta_{ab}^2) \Gamma(1 + \theta_{ab}^3)}{\Gamma(1 - \theta_{ab}^1) \Gamma(1 - \theta_{ab}^2) \Gamma(1 - \theta_{ab}^3)} \right)^{\frac{1}{2}} \tag{3.30}$$

Here S , T and U are short for $S + \bar{S}$, $T + \bar{T}$ and $U + \bar{U}$. Recall θ here depends solely on the U moduli. Converted to IIB, this expression gives

$$K_{ij}^{ab} = \delta_{ij} S^{-1/4} (T^1 T^2 T^3)^{-1/4} \prod_{I=1}^3 U_I^{-\left(\frac{1}{2} \pm \frac{1}{2} \text{sign}(I_{ab}) \theta_{ab}^1\right)} \left(\frac{\Gamma(\theta_{ab}^1) \Gamma(\theta_{ab}^2) \Gamma(1 - \theta_{ab}^1 - \theta_{ab}^2)}{\Gamma(1 - \theta_{ab}^1) \Gamma(1 - \theta_{ab}^2) \Gamma(\theta_{ab}^1 + \theta_{ab}^2)} \right)^{\frac{1}{2}} \tag{3.31}$$

The angles θ here depends only on the T moduli and the product of gamma functions has the same expansion as in eq. (3.23). This breaks the single modular weight at order

$(f/t)^2$, namely at $\mathcal{O}(\alpha'^2)$. In [71], the same expression is obtained, however without the $U_I^{-\frac{1}{2}\text{sign}(I_{ab})\theta_{ab}^I}$ term. If the term $U_I^{-\frac{1}{2}\text{sign}(I_{ab})\theta_{ab}^I}$ is present, then expanding the power it breaks the factorised form of the matter metric at $\mathcal{O}(\alpha')$, that is $\mathcal{O}(f/t)$. As $\theta_1 + \theta_2 + \theta_3 = 0$, this angular dependence of the U moduli is not present in the form of the physical Yukawa couplings.

However, the factorised structure of the matter metrics that is the fourth assumption of mirror mediation is present at leading order. The presence of a universal modular weight is certainly broken at $\mathcal{O}(\alpha'^2)$ by the gamma function product. Depending on the existence of the $U_I^{-\frac{1}{2}\text{sign}(I_{ab})\theta_{ab}^I}$, this may also be broken at $\mathcal{O}(\alpha')$.

There is a further point of interest. For toroidal examples of IBWs, the matter metrics are actually entirely diagonal and flavour-universal. This is easy to understand. The diagonal nature comes because different chiral fermions are located at different intersection points, and so are physically separated in space. The metrics for these fields can only be non-diagonal at a non-perturbative level through brane or worldsheet instantons. The flavour-universality occurs because in a torus the intersection angles are universal between different flavours: the torus is flat and two hyperplanes always intersect at the same angles. As it is the angles that determine the matter metric, it follows that on a torus the flavour-universality of the matter metrics occurs to all orders in α' (note that this does not hold for species of different gauge charges - here the brane stacks intersect at different angles, giving different matter metrics). We will see in appendix A that for Calabi-Yau intersecting brane worlds the intersection angles can be family non-universal. In this case the factorisation is expected to break at subleading order in the α' expansion.

4. Supersymmetry breaking

Section 3 has shown that the factorisation of the moduli space that is necessary to realise mirror mediation does indeed occur in string compactifications. To realise mirror mediation, it is necessary that susy breaking also respects this factorisation, with the goldstino lying dominantly in the sector mirror to that in which the flavour structure originates. While it is possible to study the moduli space in both the IIA and IIB contexts, the process of moduli stabilisation, supersymmetry breaking and hierarchy generation is much better understood for IIB compactifications. In this section we will therefore focus on type IIB models, as it is possible to be explicit about the origin of supersymmetry spontaneously broken at hierarchically low energy scales. We shall comment briefly on the IIA case at the end.

4.1 The GKP limit

In type IIB models the flavour structure arises through the complex structure moduli. If mirror mediation is to occur, supersymmetry must be dominantly broken in the Kähler moduli sector. Fortunately this is naturally realised in IIB flux compactifications. Here I shall only state results - detailed reviews of flux compactifications are in [18–21]. We start with the models of Giddings, Kachru and Polchinski [15], which correspond to flux

compactifications of IIB string theory with D3 and D7 branes on Calabi-Yau orientifolds. At leading order in g_s and α' , the low energy supergravity theory is described by

$$W = \int G_3 \wedge \Omega, \tag{4.1}$$

$$K = -2 \ln(\mathcal{V}(T_i + \bar{T}_i)) - \ln\left(i \int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S}). \tag{4.2}$$

T_i are the Kähler moduli, and Ω depends on the complex structure moduli U . For now we neglect the possible presence of additional brane moduli, but we shall remedy this below. As is well known, this theory has no-scale structure and the scalar potential reduces to

$$\begin{aligned} V &= \sum_{I,J=S,T,U} e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) \\ &= \sum_{I,J=U,S} e^K K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} = \sum_{I,J=U,S} K_{I\bar{J}} F^I \bar{F}^{\bar{J}}. \end{aligned} \tag{4.3}$$

This stabilises the dilaton and complex structure moduli in a supersymmetric fashion,

$$D_{U_i} W = D_S W = F^{U_i} = F^S = 0, \quad \forall i.$$

The Kähler moduli are unstabilised and break supersymmetry, $F^{T_j} \neq 0$. This realises the final requirement of mirror mediation, as the supersymmetry breaking occurs in one of the mirror sectors, while flavour physics is generated in the other sector.

In the GKP limit, the supersymmetry breaking actually has further structure and can be associated to the breathing mode. This can be seen by identifying the goldstino. The goldstino is the fermionic mode eaten by the massive gravitino as supersymmetry is broken. The goldstino parametrises the direction of supersymmetry breaking, and it is the couplings of this direction to matter that determine the supersymmetric soft terms.

In general the Calabi-Yau volume is a function of many moduli, $\mathcal{V} = \mathcal{V}(T_i + \bar{T}_i)$, $i = 1, \dots, h^{1,1}$. As the T_i parametrise the volume of 4-cycles, the volume is a homogeneous function of the T_i of degree $3/2$. To identify the goldstino, at any fixed point P in moduli space we can go to local coordinates. We identify and label the goldstino through the bosonic mode that the goldstino is the fermionic counterpart to. As the Kähler potential is a function only of $T_i + \bar{T}_i$ and not of the axionic components $c_i = \text{Im}(T_i)$, it is sufficient to focus on the real parts $\tau_i = \text{Re}(T_i)$ of the fields. W is independent of the T -moduli, implying

$$D_{T_j} W = \partial_{T_j} W + (\partial_{T_j} K) W = (\partial_{T_j} K) W = \frac{1}{2} (\partial_{\tau_j} K) W.$$

For any fields $\psi, \phi \in \{\tau_i\}$,

$$\partial_\psi K = -2 \frac{\partial_\psi \mathcal{V}}{\mathcal{V}}, \tag{4.4}$$

$$\partial_\psi \partial_\phi K = -2 \frac{\partial_\psi \partial_\phi \mathcal{V}}{\mathcal{V}} + 2 \frac{\partial_\psi \mathcal{V} \partial_\phi \mathcal{V}}{\mathcal{V}^2}. \tag{4.5}$$

At any point in moduli space $P \in \tau_i$ we go to local coordinates $\{\tau_i\} = \{\tau_{i,0}\} + (X, Y_i)$, in which the Y -directions are transverse to the overall volume, $\partial_{Y_i} \mathcal{V}|_P = 0$, implying that

$D_{Y_i}W = 0$. The X direction is defined to be the overall scaling direction $\tau_i \rightarrow (1 + X)\tau_i$, and so $D_XW \neq 0$. Evaluating the Kähler metric we get

$$\left(\partial_{Y_i}\partial_X V\right)\Big|_P = -2\left(\frac{\partial_{Y_i}\partial_X\mathcal{V}}{\mathcal{V}}\right)\Big|_P + 2\left(\frac{\partial_{Y_i}\mathcal{V}\partial_X\mathcal{V}}{\mathcal{V}^2}\right)\Big|_P.$$

As X is precisely the overall scaling mode $\tau_i \rightarrow (1 + X)\tau_i$ and \mathcal{V} is homogeneous in the τ_i of degree $3/2$, $\partial_X\mathcal{V} = \frac{3\mathcal{V}}{2}$. Therefore

$$\partial_{Y_i}\partial_X V\Big|_P = \frac{3}{2}\partial_{Y_i}\mathcal{V}\Big|_P = 0,$$

as the Y_i directions are defined to be transverse to the overall volume at P . It follows that $K_{XY_i}\Big|_P = 0$. This implies the off-diagonal elements of the Kähler metric vanish:

$$K = \left(\begin{array}{c|c} K_{X\bar{X}} & 0 \\ \hline 0 & K_{Y_i\bar{Y}_j} \end{array}\right).$$

As $D_{Y_i}W = 0$ and $D_XW \neq 0$, it follows that $F^X \neq 0$ and $F^{Y_i} = 0$. The X -direction thus corresponds to the goldstino - it is both orthogonal to all other directions in moduli space and the only direction to break supersymmetry. The X direction corresponded to the overall rescaling mode $\tau_i \rightarrow \lambda\tau_i$. In terms of the Calabi-Yau metric, this is simply the breathing mode

$$g_{i\bar{j}} \rightarrow \lambda^2 g_{i\bar{j}},$$

appropriately complexified with axionic fields.

The identification of the Goldstino with the breathing mode extends to the case where D3 and D7 position modes are included and there are brane moduli that mix with the Kähler and complex structure moduli. In this case the correct expression for the holomorphic chiral superfields becomes rather complicated. (cf eq. (3.8) or eq. (4.35) of [72]). The Kähler moduli no longer simply involve the complexified cycle sizes of the Calabi-Yau, but also mix with (for example) D3 position moduli, D7 position moduli and Wilson line moduli. Nonetheless the Kähler potential is still given by

$$K = -2\ln(\mathcal{V}(T + \bar{T}, \phi_{D3}, \phi_{D7}, A_{WL}, U, \dots)) - \ln\left(\int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S}),$$

where \mathcal{V} is the physical volume of the Calabi-Yau. In terms of the physical cycle sizes τ_i , the volume \mathcal{V} remains a homogeneous polynomial of degree $3/2$. However it is no longer possible to write $\tau_i = \text{Re}(T_i)$, as the complex chiral superfields have been redefined.

The argument above giving the goldstino direction nonetheless still applies. At any point in moduli space, we can go into local coordinates (X, Y_i) , where the Y -directions are transverse to the overall volume, with $\partial_{Y_i}\mathcal{V} = 0$ and thus $D_{Y_i}W = 0$. The Y_i directions now include brane motions as well as directions in Kähler moduli space. The X -direction again corresponds to the overall rescaling $\tau_i \rightarrow (1 + X)\tau_i$. As $\partial_X\mathcal{V} \propto \mathcal{V}$, we again have $K_{XY_i} = 0$. This implies the X direction is transverse to all Y directions and thus the goldstino is the breathing mode

$$\tau_i \rightarrow (1 + X)\tau_i, \quad g_{i\bar{j}} \rightarrow \mu^2 g_{i\bar{j}}.$$

This result is not so surprising, given the expression for the gravitino mass. For vanishing cosmological constant the gravitino mass is the order parameter of supersymmetry breaking. It is given by

$$m_{3/2} = e^{K/2}W = \frac{W}{\mathcal{V}}. \tag{4.6}$$

As W depends only on the complex structure moduli which have been fixed, we see that the goldstino should correspond to a rescaling of the overall volume.

The fact that the goldstino aligns with the breathing mode makes it manifest that the resulting soft masses will be flavour-universal up to possible corrections of higher order in the α' and g_s expansions. The resulting soft masses are determined by the coupling of the breathing mode to the different flavours. From the discussion in section 3, in the field theory limit the size moduli only enter the Yukawa couplings through a flavour-blind normalisation factor, characterised by the modular weight. Metric rescalings affect the normalisation, but not the structure, of the Yukawa couplings. The coupling of the goldstino to the chiral matter is therefore only sensitive to the modular weight and not to the detailed flavour structure.

The GKP models therefore give a very attractive pattern of supersymmetry breaking, which ensures that soft masses will be flavour-universal. However, as these models only stabilise the dilaton and complex structure moduli they are incomplete. We would like to retain the supersymmetry breaking structure of the GKP models while also stabilising the Kähler moduli.

In realistic phenomenological models of supersymmetry breaking, it is furthermore necessary that the gravitino be hierarchically small with a mass close to the TeV scale. From (4.6), it is clear that this can be accomplished in one of two ways: either the superpotential W is extremely small, $W \sim 10^{-15}$, or the volume \mathcal{V} is extremely large $\mathcal{V} \sim 10^{15}$. We now consider moduli-stabilised cases where these properties are realised. These cases correspond respectively to the KKLT and large volume frameworks for moduli-stabilisation.

4.2 Models with full moduli stabilisation

We consider two proposals for low energy supersymmetry together with moduli stabilisation in IIB flux models.

4.2.1 KKLT

In KKLT [16], non-perturbative effects (from either Euclidean D3-instantons or gaugino condensation) are introduced in each Kähler modulus. In the low energy theory these induce a superpotential

$$W = W_0 + \sum_i A_i e^{-a_i T_i}. \tag{4.7}$$

A hierarchically small gravitino mass, required to stabilise the gauge hierarchy, can be obtained by tuning the tree-level flux superpotential W_0 to very small values. At the current level of understanding there is no dynamical explanation for why the flux superpotential should be small rather than taking $\mathcal{O}(1)$ values. The Kähler moduli are stabilised supersymmetrically by solving the equations $D_{T_i}W = 0$. In a supersymmetric vacuum all the

soft terms vanish and the vacuum energy is AdS, with $F^U = F^T = 0$ and

$$V_0 = -3m_{3/2}^2 M_P^2.$$

As the AdS vacuum is supersymmetric, this step destroys the factorised structure of supersymmetry breaking present in GKP. To match the cosmological constant it is necessary to include a source of susy-breaking energy to uplift the AdS vacuum to Minkowski. The resulting phenomenology of supersymmetry breaking is determined entirely by the origin and nature of this uplifting, and different methods can give quite different results.

Generally, we imagine uplifting with an additional hidden sector. The hidden sector can arise from anti-D3 branes, fluxes or strong gauge dynamics. At the level of the scalar potential for the Kähler moduli, the precise origin is not so important and the uplifting can be parametrised by adding a term

$$V = \sum_{i=T} e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) + \frac{\epsilon}{\mathcal{V}^2}.$$

The dependence on the volume comes from the universal e^K term in the scalar potential. In certain cases ϵ may also depend on the volume; for example when uplifting is performed using a warped throat then ϵ has an effective dependence of $\mathcal{V}^{2/3}$, giving an overall dependence of $\mathcal{V}^{-4/3}$. So long as such an uplift term depends solely on the volume, directions Y transverse to the volume will still be extremised at $D_Y W = 0$, and restricting to the Kähler moduli alone, the dominant F-term will still align with the overall breathing mode. However it is easy to check [26] that the magnitude of the resulting F-term for the T fields is $F^T \sim \frac{m_{3/2}}{\ln(m_{3/2})}$. As

$$(F^T)^2 \sim \frac{m_{3/2}^2}{\ln(M_P/m_{3/2})^2} \ll 3m_{3/2}^2 M_P^2,$$

this implies the T -fields only ever play a subdominant role in susy breaking, in contrast to the GKP case. It is therefore not possible for KKLT models to realise mirror mediation in its pure sense, as the T fields can never dominate supersymmetry breaking.

The soft terms in a KKLT framework are determined by the details and couplings of the uplifting sector. For example, suppose the uplifting is carried out using a metastable susy breaking vacuum in the complex structure moduli sector as proposed in [73]. In this case the dominant F-terms are located in the complex structure moduli sector, giving

$$F^U \sim m_{3/2}, \quad F^T \sim \frac{m_{3/2}}{\ln m_{3/2}}.$$

This is bad news from the viewpoint of flavour physics, as the dominant F-term lies in precisely the same sector from which flavour physics originates. This is expected to lead to large unobserved FCNCs and CP violation, generated for example through the $F^U \partial_U Y_{\alpha\beta\gamma}(U)$ contribution to the A-terms. The flavour constraints may be mitigated through the large scalar masses, but this is at tension with the requirement of naturalness in the Higgs sector. Similar expressions apply for uplifting with a matter sector.

If the matter metrics of the Standard Model fields do not depend on the hidden sector fields, then the F^{hidden} terms do not contribute to scalar masses. In this case the scalar

masses come solely from the universal $m_{3/2}^2$ supergravity term in (2.9). In this case the soft scalar masses are heavier than the gauginos by a factor $\ln(M_P/m_{3/2})$,

$$m_i^2 \simeq m_{3/2}^2, \quad M_i \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})}.$$

The soft masses are now so heavy that flavour constraints are less significant. However, given the direct search limits on gaugino masses, the heaviness of the scalar masses is in tension with naturalness in the Higgs sector.

In order to satisfy the flavour constraints with scalar and gaugino masses of comparable order, it is necessary that the supersymmetry breaking that cancels the vacuum energy be both insensitive to CP and flavour and also cancel the universal supergravity $m_{3/2}^2$ contribution to the scalar masses. This is equivalent to the statement that the uplifting supersymmetry breaking is sequestered from the visible sector. If this occurs, both scalar and gaugino masses are dominantly determined by the F^T components, together with anomaly-mediated contributions that are of size

$$m_{\text{anomaly}} \sim \frac{m_{3/2}}{8\pi^2} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})}.$$

This is the mirage mediation scenario [30–32].

4.2.2 LARGE volume models

KKLT stabilisation only incorporates non-perturbative superpotential corrections to the GKP framework. The additional inclusion of perturbative α' corrections to the Kähler potential gives rise to the large volume models [17, 33]. Somewhat unexpectedly, the inclusion of α' corrections to the scalar potential generates a new supersymmetry-breaking minimum at exponentially large volumes. The reason why α' corrections can affect the potential at such large volumes is that the tree-level potential vanishes due to the no-scale structure, making the α' corrections the leading perturbative contributions to the scalar potential. These minima exist for all values of the flux superpotential W_0 . In addition to improved technical control, the appearance of exponentially large volumes gives a dynamical generation of the weak hierarchy.

The simplest example of these models is for the $\mathbb{P}_{[1,1,1,6,9]}^4$ Calabi-Yau, the volume of which is $\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$. The supergravity theory is

$$K = -2 \ln \left(\left(\frac{T_b + \bar{T}_b}{2} \right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2} \right)^{3/2} + \frac{\xi}{g_s^{3/2}} \right), \quad (4.8)$$

$$W = W_0 + A_s e^{-a_s T_s}. \quad (4.9)$$

ξ parametrises the α' correction. This theory can be shown to have a supersymmetry breaking minimum at $\mathcal{V} \sim W_0 e^{a_s \tau_s}$, with $\tau_s \sim \xi^{2/3}/g_s$. There are two cycles, one large (τ_b) and one small (τ_s). The large cycle controls the overall volume and the small cycle the size of a blow-up mode. The standard model is localised on D7 branes wrapping the blow-up

mode. The phenomenology of this model with regard to low-energy supersymmetry has been studied in [33, 34, 36, 40, 51, 50].

Before uplifting, the AdS vacuum energy is

$$V_{\text{AdS}} \sim m_{3/2}^3 M_P \ll m_{3/2}^2 M_P^2.$$

The fact that V_{AdS} is hierarchically smaller than the natural supergravity scale of $m_{3/2}^2 M_P^2$ indicates that the no-scale structure present in GKP survives to leading order: the vacuum energy is much less than the scale of susy breaking would suggest. This can also be seen in the mass of the volume modulus, which is [33]

$$m_{\text{Vol}} \sim m_{3/2} \left(\frac{m_{3/2}}{M_P} \right)^{\frac{1}{2}} \ll m_{3/2}.$$

The lightness of the volume modulus and the smallness of the vacuum energy are both residues of the tree-level no-scale structure. The volume modulus is the pseudo-Goldstone boson of the tree-level scaling symmetry $g \rightarrow \lambda g$, which is broken by α' corrections. The F-terms are [33, 40]

$$F^b \sim m_{3/2} M_P, \quad F^s \sim m_{3/2}^{\frac{3}{2}} M_P^{\frac{1}{2}}.$$

The goldstino aligns dominantly with the overall breathing mode and the susy breaking of the large volume models is largely inherited from that of the GKP case.

Additional sources of supersymmetry breaking are necessary to cancel the negative vacuum energy. The required magnitude of this is $(F^{\text{uplift}})^2 \sim m_{3/2}^3 M_P \ll (F^T)^2 \sim m_{3/2}^2 M_P^2$, and so the magnitude of soft terms due to the uplifting energy is

$$m_{\text{soft}} \sim \frac{F^{\text{uplift}}}{M_P} \sim m_{3/2} \left(\frac{m_{3/2}}{M_P} \right)^{\frac{1}{2}}.$$

For $m_{3/2} \sim 1\text{TeV}$, $m_{\text{soft}} \sim 1\text{MeV}$ and so this contribution is negligible.

Independent of the details of the uplift, the goldstino lies dominantly in the Kähler moduli sector. The large volume models thus realise mirror mediation, as the supersymmetry breaking is always dominated by the Kähler moduli. The goldstino is, up to a small correction of order $\mathcal{V}^{-1/2}$, still aligned with the overall breathing mode. Matter is localised on D7 branes wrapping the blow-up cycle, which can be thought of as a hole in the bulk of the Calabi-Yau. The goldstino corresponds to a local metric rescaling (the hole growing into the bulk).

5. Conclusions

The requirement of no new large contributions to flavour-changing neutral currents or CP violation is one of the strongest constraints on the MSSM Lagrangian. This article has investigated when supersymmetry breaking in string theory compactifications generates flavour-universal soft terms. It has identified a set of sufficient conditions for flavour universality. These set of conditions were called ‘mirror mediation’. They correspond to the

factorisation of the hidden sector into two classes of field, with one class sourcing flavour physics and the mirror class responsible for susy breaking.

This factorisation is artificial on the level of effective field theory, but naturally occurs within the effective actions that apply in string compactifications. In this case the sectors can be identified with the two mirror sectors of geometric moduli associated with Calabi-Yau geometries, the Kähler and complex structure moduli. At leading order these sectors are decoupled and do not mix. In both IIA and IIB compactifications the combination of holomorphy and shift symmetries implies that the superpotential Yukawa couplings can depend only on one class of moduli. In IIB compactifications flavour physics comes from the complex structure moduli while for IIA models flavour physics originates with the Kähler moduli. This was illustrated through the explicit formulae for the Yukawa couplings and matter field kinetic terms in type II compactifications. The factorisation is broken at higher order in g_s and α' . This breaking is suppressed at large volume and weak coupling.

Mirror mediation requires that supersymmetry breaking dominantly occurs in the mirror sector to that in which flavour physics originates. In IIB models this implies that supersymmetry breaking should occur in the Kähler moduli sector. This is realised by GKP flux compactifications, which have no-scale structure. In this case the goldstino was shown to align exactly with the overall breathing mode. Going to the case of full moduli stabilisation, we considered the large volume and KKLT models. In the large volume case, the structure of supersymmetry breaking is largely inherited from the GKP case. The goldstino aligns dominantly with the overall volume and susy breaking occurs in the Kähler moduli sector. The large volume models therefore give an explicit realisation of the full mirror mediation structure. In KKLT-based models, the dominant susy breaking comes from the hidden sector used in uplifting while susy breaking in the T-sector is subdominant. The structure of the soft terms depends on the details of the hidden sector and is model-dependent.

There are several directions for future work. First, in any $\mathcal{N} = 1$ compactification the factorisation of the moduli space will be broken at subleading order, corresponding to loop effects in either the α' or g_s expansions. Such breaking of factorisation, while suppressed by loop factors, has the potential to lead to breaking of flavour universality in the soft terms. The actual magnitude of corrections to universality have large phenomenological consequences, and it would be very interesting to quantify this in specific models. Secondly, the structure of mirror mediation suggests that in phenomenological models of IIA string theory supersymmetry breaking should dominantly occur in the complex structure moduli sector, while the Kähler moduli should be stabilised supersymmetrically. It would be interesting to build fully stabilised models where this occurs, together with the generation of hierarchically low supersymmetry breaking scales.

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A. Intersection angles in Calabi-Yau backgrounds

In toroidal models, the three angles that characterise D6-brane intersections are independent of flavour: repeated intersections of the same two stacks occur with the same angles (however note that for fields of different gauge charges the intersection angles vary). The purpose of the appendix is to investigate whether this still holds in Calabi-Yau cases: we will find that for Calabi-Yau models the intersection angles are expected to be flavour non-universal, even for fields of the same gauge charge.

The candidates for supersymmetric branes in IIA are D4, D6 and D8 branes. A Calabi-Yau has no 1- or 5-cycles, so this focuses our attention on D6-branes.¹⁰ The Calabi-Yau breaks the $\mathcal{N} = 8$ supersymmetry of type II string theory to $\mathcal{N} = 2$. A supersymmetric D-brane embedding further breaks this to $\mathcal{N} = 1$. Considering a single D6-brane, the condition that the embedding is supersymmetric is that the brane wraps a *special Lagrangian* cycle. A *Lagrangian* 3-cycle Σ is one for which the pullback of the Kähler form vanishes $J|_{\Sigma} = 0$, i.e.

$$J(\mathbf{e}_i, \mathbf{e}_j) = 0, \tag{A.1}$$

for all $\mathbf{e}_i, \mathbf{e}_j$ lying in the brane worldvolume Σ . A *special Lagrangian* 3-cycle is one which is volume-minimising, i.e. is calibrated by the holomorphic (3,0) form Ω ,

$$\text{Im}(e^{i\theta}\Omega)|_{\Sigma} = 0, \quad \text{Re}(e^{i\theta}\Omega)|_{\Sigma} = \text{vol}_{\Sigma}, \tag{A.2}$$

for some phase θ . The brane breaks the $\mathcal{N} = 2$ supersymmetry of the Calabi-Yau to $\mathcal{N} = 1$. In terms of the $\mathcal{N} = 2$ supercharges ψ_1 and ψ_2 , the supercharge ψ preserved by the D-brane is $\psi = (\cos \theta) \psi_1 + (\sin \theta) \psi_2$.

D-branes carry positive charge (+Q) and tension (+Q). To avoid a global RR tadpole while still preserving supersymmetry, it is necessary to introduce objects that can simultaneously carry both negative tension and charge. Such objects are orientifold O6-planes. These are located at fixed point sets of the orientifold action. They also wrap special Lagrangian cycles and break the $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$. In a globally $\mathcal{N} = 1$ supersymmetric configuration, all D-branes must preserve the same supercharge as the O-planes and so must be calibrated with the same angle θ .

As 3-dimensional objects in a 6-dimensional space, D6 branes generically have pointlike intersections. If stacks of N and M branes intersect, then at the intersection locus chiral matter is localised in the (N, \bar{M}) representation. In intersecting brane worlds, the different gauge groups correspond to different brane stacks. Family replication is due to nonvanishing intersection numbers for distinct special Lagrangian 3-cycles. The number of chiral families is given by the topological intersection number of distinct cycles. The physics of flavour is equivalent to the physics of the different intersections.

The intersection of two branes is always characterised by three intersection angles. Each intersection is specified by angles θ_i , $i = 1 \dots 3$. This is illustrated in figure 1, which shows a brane embedded in local complex coordinates z_1, z_2, z_3 with embedding $\text{Im}(z_i) = 0$. It intersects with another brane embedded as $\text{Im}(e^{-i\theta_i} z_i) = 0$ at an intersection locus $z_i = 0, i = 1, 2, 3$ with intersection angles θ_1, θ_2 and θ_3 .

¹⁰although note supersymmetric coisotropic D8 branes can exist [74].

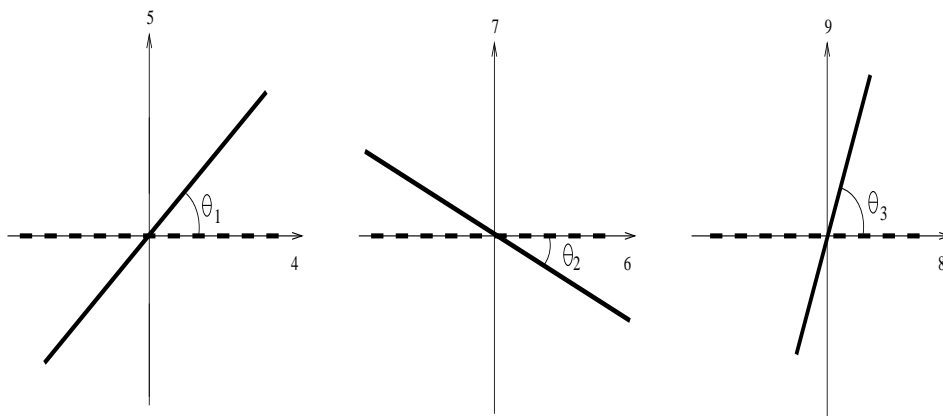


Figure 1: A locally factorisable brane intersection.

Any intersection is invariantly characterised by three angles. These angles are the eigenvalues of the $SU(3)$ matrix that locally transforms one sLag into another. That is, if the first sLag corresponds to $\text{Re}(z_i) = 0$, and the second sLag corresponds to $\text{Re}(w_i) = 0$, where now the intersection point $w_i = \mathcal{M}_{ij}z_j$ with $\mathcal{M} \in SU(3)$, the intersection angles $\theta_1, \theta_2, \theta_3$ are given by the eigenvalues of \mathcal{M}_{ij} .

For the case of sLags intersecting in a Calabi-Yau, we do not need to know the metric or the Kähler form in order to compute the intersection angles. We here focus on a four-dimensional subspace, parametrised by coordinates x_1, x_2, x_3, x_4 , with $z_1 = x_1 + ix_2$ and $z_2 = x_3 + ix_4$. We can write

$$\begin{aligned}
 g_{z_i z_j} &= \frac{\partial x^\alpha}{\partial z^i} \frac{\partial x^\beta}{\partial z^j} g_{\alpha\beta} \\
 g_{z_i \bar{z}_j} &= \frac{\partial x^\alpha}{\partial z^i} \frac{\partial x^\beta}{\partial \bar{z}^j} g_{\alpha\beta} \\
 g_{\bar{z}_i \bar{z}_j} &= \frac{\partial x^\alpha}{\partial \bar{z}^i} \frac{\partial x^\beta}{\partial \bar{z}^j} g_{\alpha\beta}
 \end{aligned}
 \tag{A.3}$$

In terms of the real metric $g_{\alpha\beta}$, we therefore have

$$\begin{aligned}
 g_{z_1 z_1} &= \frac{1}{4}(g_{11} - g_{22}) - \frac{i}{2}g_{12}, \\
 g_{z_1 \bar{z}_1} &= \frac{1}{4}(g_{11} + g_{22}), \\
 g_{\bar{z}_1 \bar{z}_1} &= \frac{1}{4}(g_{11} - g_{22}) + \frac{i}{2}g_{12},
 \end{aligned}
 \tag{A.4}$$

and likewise for $g_{z_2 z_2}$. We also have

$$\begin{aligned}
 g_{z_1 \bar{z}_2} &= \frac{1}{4}(g_{13} + g_{24}) + \frac{i}{4}(g_{14} - g_{23}), \\
 g_{z_2 \bar{z}_1} &= \frac{1}{4}(g_{13} + g_{24}) - \frac{i}{4}(g_{14} - g_{23}), \\
 g_{z_2 \bar{z}_1} &= (g_{z_1 \bar{z}_2})^*,
 \end{aligned}$$

$$\begin{aligned}
 g_{z_1 z_2} &= \frac{1}{4}(g_{13} - g_{24}) - \frac{i}{4}(g_{14} + g_{23}) \\
 g_{\bar{z}_1 \bar{z}_2} &= \frac{1}{4}(g_{13} - g_{24}) + \frac{i}{4}(g_{14} + g_{23}).
 \end{aligned}
 \tag{A.5}$$

Fixing the complex structure is equivalent to requiring $g_{z_i z_j} = 0$, which imposes the conditions

$$g_{11} = g_{22}, \quad g_{33} = g_{44}, \quad g_{12} = g_{34} = 0, \quad g_{13} = g_{24}, \quad g_{14} = -g_{23}.$$

Any variations of the Kähler class which leaves the complex structure unaltered must satisfy the above conditions at the intersection locus. We can write out the Kähler form as

$$iJ = g_{11} dx^1 \wedge dx^2 + g_{33} dx^3 \wedge dx^4 + g_{23} (dx^1 \wedge dx^3 + dx^2 \wedge dx^4) + g_{13} (dx^1 \wedge dx^4 - dx^2 \wedge dx^3) \tag{A.6}$$

This is the general local form of the Kähler class at $z = 0$ that preserves the complex structure.

The fact that a brane wrapping the cycle $x_2 = x_4 = 0$ is special Lagrangian forces $g_{23} = 0$ along the worldvolume of this brane, as otherwise the pull-back of the Kähler form would not vanish. As the intersecting brane is also Lagrangian, we also require the pull-back of the Kähler form onto its worldvolume to vanish. At the intersection locus, this implies that $g_{13} = 0$.

The fact that the two intersecting branes are wrapping Lagrangian cycles therefore restricts the locally non-vanishing components of the metric to $g_{11} = g_{22}$ and $g_{33} = g_{44}$. From the geometry of the intersection, it is clear that for branes wrapping Lagrangian cycles the local intersection angles can be determined independently of the local value of the Kähler form at the intersection point.

Writing out $\Omega = (dx^1 + idy^1) \wedge (dx^2 + idy^2) \wedge (dx^3 + idy^3)$, it is easy to see that the special Lagrangian condition takes the well-known form

$$\theta_1 + \theta_2 + \theta_3 = 0. \tag{A.7}$$

Unlike for D6-branes, IIB intersections are not pointlike and so it is not obvious that the chiral matter metrics can be characterised by three angles. However mirror symmetry suggests that this is the case, and we assume here that the expression (A.7) continues to hold for IIB IBWs on a Calabi-Yau.

The quintic. The simplest Calabi-Yau is the Fermat quintic. This is given by the hypersurface in \mathbb{P}^4

$$\sum z_i^5 = 0. \tag{A.8}$$

Special Lagrangian 3-cycles on the quintic have been studied in [75–78]. There exist a class of 625 sLag cycles, described by

$$|k_1, k_2, k_3, k_4, k_5\rangle = \{z_i : \text{Re}(\omega^{k_1 z_1}) = \text{Re}(\omega^{k_2 z_2}) = \dots = \text{Re}(\omega^{k_5 z_5}) = 0.\}$$

Here $\omega^5 = 1$ and we may take $\omega = e^{2\pi i/5}$. Due to the \mathbb{P}^4 identification $(z_1, z_2, z_3, z_4, z_5) \equiv \lambda(z_1, z_2, z_3, z_4, z_5)$, the cycles $|k_1, k_2, k_3, k_4, k_5\rangle$ and $|k_1 + 1, k_2 + 1, k_3 + 1, k_4 + 1, k_5 + 1\rangle$ are identified, and this family of sLags only contains 625 distinct examples.

The topological intersection matrix of these sLags was computed in [78]. The intersection number of the cycles $|1, 1, 1, 1, 1\rangle$ and $|k_1, k_2, k_3, k_4, k_5\rangle$ is given by the coefficient of $g_1^{k_1} g_2^{k_2} g_3^{k_3} g_4^{k_4} g_5^{k_5}$ in

$$\prod_{i=1}^5 (g_i + g_i^2 - g_i^3 - g_i^4) / \left\{ g_i^5 \equiv 1 \forall i, g_1 g_2 g_3 g_4 g_5 \equiv 1 \right\}$$

In general the cycle $|k_1, k_2, k_3, k_4, k_5\rangle$ is calibrated by the form $\omega^{k_1+k_2+k_3+k_4+k_5} \Omega$ and so cycles are only mutually sLag if $k_1 + k_2 + k_3 + k_4 + k_5 = 0 \pmod{5}$. There are then 5 sets of 125 mutually sLag - i.e. mutually supersymmetric - cycles. It turns out that for these sets of cycles the intersection matrix vanishes: no supersymmetric chiral matter can exist.

Other weighted \mathbb{P}^4 s. The analysis for the quintic can be extended to other Calabi-Yaus that can be written as a hypersurface in a weighted projective space. These will in fact allow sLags to intersect with non-zero topological intersection numbers.

The weighted projective space $\mathbb{P}_{[a,b,c,d,e]}^4$ is defined by

$$(z_1, z_2, z_3, z_4, z_5) \sim (\lambda^a z_1, \lambda^b z_2, \lambda^c z_3, \lambda^d z_4, \lambda^e z_5).$$

The degree $D = (a + b + c + d + e)$ hypersurface in $\mathbb{P}_{[a,b,c,d,e]}^4$ satisfies the condition that $c_1(\mathcal{M}) = 0$ [79] and therefore admits a Calabi-Yau metric. Up to non-polynomial deformations, the complex structure moduli space is described by the space of inequivalent homogeneous polynomials of degree D . If a, b, c, d, e all divide D , then the moduli space contains a Fermat hypersurface. This hypersurface can be written as

$$z_1^{n_1} + z_2^{n_2} + z_3^{n_3} + z_4^{n_4} + z_5^{n_5} = 0. \tag{A.9}$$

Here $n_1 = D/a, n_2 = D/b, \dots, n_5 = D/e$. At the Fermat locus an enhanced $\mathbb{Z}^{n_2} \otimes \mathbb{Z}^{n_3} \otimes \mathbb{Z}^{n_4} \otimes \mathbb{Z}^{n_5}$ discrete symmetry exists, corresponding to coordinate rotations. Special Lagrangian cycles can be constructed for these Calabi-Yaus in analogy to the construction for the quintic. The Fermat hypersurface admits an antiholomorphic involution $z \rightarrow \bar{z}$. This involution has as its fixed point the fundamental sLag, which we denote by $|0, 0, 0, 0, 0\rangle$. This sLag is given by

$$(x_1, x_2, x_3, x_4, x_5) \text{ with } x_1^{D/a} + x_2^{D/b} + x_3^{D/c} + x_4^{D/d} + x_5^{D/e} = 0.$$

We can construct a family of sLags by acting with the discrete symmetry generators on the fundamental sLag. We denote by $|\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\rangle$ the sLag that arises by rotating the fundamental sLag by the following discrete transformation

$$\begin{aligned} z_1 &\rightarrow e^{2\pi i \lambda_1 a/D} z_1, \\ z_2 &\rightarrow e^{2\pi i \lambda_2 b/D} z_2, \\ z_3 &\rightarrow e^{2\pi i \lambda_3 c/D} z_3, \\ z_4 &\rightarrow e^{2\pi i \lambda_4 d/D} z_4, \\ z_5 &\rightarrow e^{2\pi i \lambda_5 e/D} z_5. \end{aligned} \tag{A.10}$$

In cases where some of the a, \dots, e are even, further families of sLags not included in (A.10) may be found using half-integral actions of the discrete symmetry generators; for example, when n_1 is even this corresponds to the fixed point set under the involution

$$\begin{aligned} z_1 &\rightarrow \exp 2\pi i/n_1 z_1, \\ z_2 &\rightarrow \bar{z}_2, \\ z_5 &\rightarrow \bar{z}_5. \end{aligned}$$

This is familiar from IIA toroidal orientifolds, where for even orientifold actions there exist distinct O-planes that are not related by the orbifold action. Using the above procedure several families of sLags may be generated for the weighted projective spaces. As the cycles are known explicitly, the intersection numbers may be computed using geometric means, by explicitly locating the intersection points. Through examination of the local intersection it is possible to compute the topological intersection number, counting the sign of each intersection.

Mutually supersymmetric sLags must be calibrated by the same holomorphic 3-form. From the definition of Ω ,

$$\Omega = \frac{1}{2\pi i} \int \frac{z_5 dz_1 \wedge dz_2 \wedge dz_3 \wedge dz_4}{p(z)},$$

it follows that $|\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \rangle$ is calibrated by the same form as $|0, 0, 0, 0, 0 \rangle$ as long as $a\lambda_1 + b\lambda_2 + c\lambda_3 + d\lambda_4 + e\lambda_5 = 0 \pmod{D}$.

Using the above construction of sLags for general weighted projective spaces, we can investigate the intersection properties of sLags beyond the case of the quintic. We have considered all weighted projective spaces where a, \dots, e are all odd. These are

$$\mathbb{P}^4_{[1,1,1,1,1]}, \quad \mathbb{P}^4_{[1,1,1,3,3]}, \quad \mathbb{P}^4_{[1,3,3,7,7]}, \quad \mathbb{P}^4_{[1,1,3,5,5]}, \quad \mathbb{P}^4_{[1,3,3,3,5]}, \quad \mathbb{P}^4_{[1,5,9,15,15]}.$$

From this we can conclude that for generic mutually supersymmetric sLags with multiple chiral intersections the intersection angles differ between intersections.

As an example, we consider the Calabi-Yau hypersurface in $\mathbb{P}^4_{[1,3,3,3,5]}$. This space is defined by

$$(z_1, z_2, z_3, z_4, z_5) \sim (z_1, \lambda^3 z_2, \lambda^3 z_3, \lambda^3 z_4, \lambda^5 z_5).$$

The Fermat hypersurface is given by

$$z_1^{15} + z_2^5 + z_3^5 + z_4^5 + z_5^3 = 0, \tag{A.11}$$

and admits a discrete $\mathbb{Z}^5 \otimes \mathbb{Z}^5 \otimes \mathbb{Z}^5 \otimes \mathbb{Z}^3$ symmetry. The fundamental involution is $z_i \rightarrow \bar{z}_i$ and has a fixed point set $(\lambda x_1, \lambda^3 x_2, \lambda^3 x_3, \lambda^3 x_4, \lambda^5 x_5)$ with $x \in \mathbb{R}$ and

$$x_1^{15} + x_2^5 + x_3^5 + x_4^5 + x_5^3 = 0.$$

This has a unique solution for x_5 given x_1, \dots, x_4 and the fixed point set is topologically an \mathbb{RP}^3 . We denote this fundamental sLag by $|0, 0, 0, 0, 0 \rangle$.

Now consider the sLag $|0, 1, 1, 3, 0 \rangle$. This is obtained by acting on the fundamental sLag with

$$z_1 \rightarrow z_1, z_2 \rightarrow e^{2\pi i/5} z_2, z_3 \rightarrow e^{2\pi i/5} z_3, z_4 \rightarrow e^{6\pi i/5} z_4, z_5 \rightarrow z_5$$

It has fixed point set

$$(\lambda x_1, \lambda^3 e^{2\pi i/5} x_2, \lambda^3 e^{2\pi i/5} x_3, \lambda^3 e^{6\pi i/5} x_4, \lambda^5 x_5)$$

with $x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0$.

The sLags $|0, 0, 0, 0, 0 \rangle$ and $|0, 1, 1, 3, 0 \rangle$ intersect at the following loci:

$$(1, 0, 0, 0, -1)$$

and

$$(0, 0, 0, 1, -1) \equiv (0, 0, 0, e^{6\pi i/5}, -1).$$

For the first intersection, we can go to local complex coordinates (w_1, w_2, w_3) defined by

$$(1, w_1, w_2, w_3, (-1 + w_1^5 + w_2^5 + w_3^5)^{1/3}).$$

In these coordinates the sLag $|0, 0, 0, 0, 0 \rangle$ is described by

$$\text{Re}(w_1) = \text{Re}(w_2) = \text{Re}(w_3) = 0$$

while the sLag $|0, 1, 1, 3, 0 \rangle$ is described by

$$\text{Re}(e^{-2\pi i/5} w_1) = \text{Re}(e^{-2\pi i/5} w_2) = \text{Re}(e^{-6\pi i/5} w_3).$$

From this we can deduce the local intersection angles at $(1, 0, 0, 0, -1)$ to be $(2\pi/5, 2\pi/5, 6\pi/5)$.

For the second intersection, we can define local complex coordinates (w_1, w_2, w_3) by

$$(w_1, w_2, w_3, 1, (-1 + w_1^5 + w_2^5 + w_3^5)^{1/3}).$$

In these coordinates the sLag $|0, 0, 0, 0, 0 \rangle$ is described by

$$\text{Re}(w_1) = \text{Re}(w_2) = \text{Re}(w_3) = 0$$

and the sLag $|0, 1, 1, 3, 0 \rangle$ is described by

$$\text{Re}(e^{2\pi i/5} w_1) = \text{Re}(e^{4\pi i/5} w_2) = \text{Re}(e^{4\pi i/5} w_3).$$

From this we can deduce the local intersection angles to be $(-2\pi/5, -4\pi/5, -4\pi/5)$.

The sLags $|0, 0, 0, 0, 0 \rangle$ and $|0, 1, 1, 3, 0 \rangle$ also intersect along the S^1 $(0, x_2, x_3, 0, x_5)$. The normal bundle of a sLag is equivalent to its tangent bundle. As the tangent bundle of an S^1 admits a global non-vanishing section, there likewise admits a global non-vanishing section of the normal bundle, and so this intersection is not topological; it can be removed by a deformation of the cycles.

The sign of each intersection is given by $\prod_i \text{sgn}(e^{i\theta_i})$, where θ_i are the three intersection angles. The sign of each intersection above is -1 and so the topological intersection number of the above two cycles is -2 . This shows that for Calabi-Yau intersecting brane worlds, there does not seem to be a reason for the intersection angles of two brane stacks to be family-universal: fields of the same charges but different families can have different intersection angles.

Similar behaviour can be seen for sLags in $\mathbb{P}^4_{[1,1,1,3,3]}$ and $\mathbb{P}^4_{[1,1,3,5,5]}$. In the former case, the intersection of $|0,0,0,0,0\rangle$ and $|0,0,3,1,1\rangle$ gives family non-universal intersection angles, whereas in the latter cases non-universality is found for the intersections of $|0,0,0,0,0\rangle$ with the sLags $|0,1,3,1,0\rangle$, $|0,2,1,2,0\rangle$ and $|0,4,2,1,0\rangle$.

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